Marking Scheme Strictly Confidential

(For Internal and Restricted use only) Senior Secondary Examination, 2026

SUBJECT NAME MATHEMATICS (Q.P. CODE – 65/1/1)

<u>General Instructions: -</u>	
1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and BNS."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left- hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.

10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - • Leaving answer or part thereof unassessed in an answer book. • Giving more marks for an answer than assigned to it. • Wrong totalling of marks awarded on an answer. • Wrong transfer of marks from the inside pages of the answer book to the title page. • Wrong question wise totalling on the title page. • Wrong totalling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) • Half or a part of the answer marked correct and the rest as wrong, but no marks awarded.
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the “Guidelines for Spot Evaluation” before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totalled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME

MATHEMATICS (Subject Code–041)

(PAPER CODE: 65/1/1)

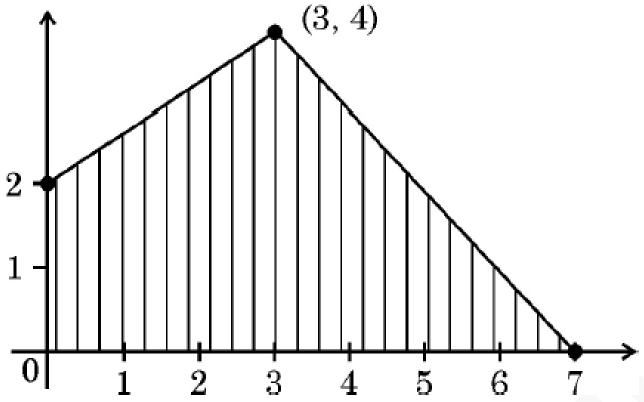
Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	If $2\cos^{-1}x = y$, then (A) $0 \le y \le \pi$ (B) $-\pi \le y \le \pi$ (C) $0 \le y \le 2\pi$ (D) $-\pi \le y \le 0$		
Sol.	(C) $0 \le y \le 2\pi$		1
2.	Which of the following cannot be the order of a row-matrix ? (A) 2×1 (B) 1×2 (C) 1×1 (D) $1 \times n$		
Sol.	(A) 2×1		1
3.	Which of the following properties is/are true for two matrices of suitable orders ? (i) $(A + B)' = A' + B'$ (ii) $(A - B)' = B' - A'$ (iii) $(AB)' = A'B'$ (iv) $(kAB)' = kB'A'$ (k is a scalar) (A) (i) only (B) (i), (ii) and (iii) (C) (i) and (ii) (D) (i) and (iv)		
Sol.	(D) (i) and (iv)		1

4.	If $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$, then (A) $\Delta_1 = 2\Delta_2$ (B) $\Delta_2 = -2\Delta_1$ (C) $\Delta_1 = \Delta_2$ (D) $\Delta_2 = -\Delta_1$		
Sol.	(B) $\Delta_2 = -2\Delta_1$		1
5.	One of the values of x for which $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is (A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$		
Sol.	(B) $\frac{\pi}{4}$		1
6.	If A and B are skew symmetric matrices of same order, then which of the following matrices is also skew symmetric ? (A) AB (B) AB + BA (C) $(A + B)^2$ (D) A – B		
Sol.	(D) A – B		1

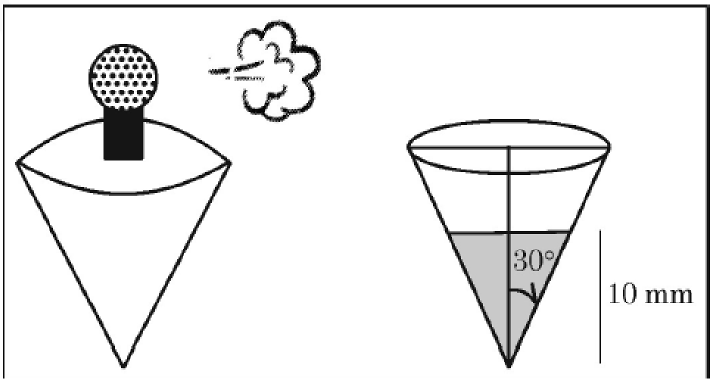
7.	The least value of $f(x) = x^3 - 12x$, $x \in [0, 3]$ is (A) -16 (B) -9 (C) 0 (D) 16		
Sol.	(A) -16		1
8.	If $\int \frac{3ax}{b^2 + c^2x^2}\,dx = A \log \left\| b^2 + c^2x^2 \right\| + K$, then the value of A is (A) $3a$ (B) $\frac{3a}{2b^2}$ (C) $\frac{3a}{b^2c^2}$ (D) $\frac{3a}{2c^2}$		
Sol.	(D) $\frac{3a}{2c^2}$		1
9.	The value of $\int_{-1}^{1} \frac{x^3}{x^2 + 2\|x\| + 1}\,dx$ is (A) 0 (B) $\log 2$ (C) $2 \log 2$ (D) $\frac{1}{2} \log 2$		
Sol.	(A) 0		1

10.	The area bounded by the curve $y = x\|x\|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by (A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$		
Sol.	(C) $\frac{2}{3}$		1
11.	The integrating factor of differential equation $R\frac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is (A) $e^{\int \frac{P}{Q} dy}$ (B) $e^{\int P dy}$ (C) $e^{\int \frac{P}{R} dy}$ (D) $e^{\int \frac{P}{R} dx}$		
Sol.	(C) $e^{\int \frac{P}{R} dy}$		1
12.	The order and degree of the differential equation $\frac{d}{dx}(e^y) = 0$ respectively are (A) 0, 1 (B) 1, 1 (C) 2, 1 (D) 1, not defined		
Sol.	(B) 1, 1		1

13.	The value of p for which vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is (A) 0 (B) 1 (C) $\frac{5}{2}$ (D) $-\frac{5}{2}$		
Sol.	(C) $\frac{5}{2}$		1
14.	The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is (A) 8 (B) –8 (C) 2 (D) $\frac{5}{2}$		
Sol.	(A) 8		1
15.	If $\|\vec{a}\| = 8$, $\|\vec{b}\| = 3$ and $\|\vec{a} \times \vec{b}\| = 12$, then the value of $\|\vec{a} \cdot \vec{b}\|$ (A) $6\sqrt{3}$ (B) $8\sqrt{3}$ (C) $12\sqrt{3}$ (D) $3\sqrt{12}$		
Sol.	(C) $12\sqrt{3}$		1
16.	The length of perpendicular drawn from point (2, 5, 7) on line $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ is (A) 2 (B) 5 (C) $\sqrt{74}$ (D) $\sqrt{78}$		
Sol.	(C) $\sqrt{74}$		1

17.	The feasible region of a linear programming problem with objective function $Z = 5x + 7y$ is shown below :  The maximum value of Z – minimum value of Z is (A) 8 (B) 29 (C) 35 (D) 43		
Sol.	(D) 43		1
18.	The degree of an objective function of a linear programming problem is (A) 0 (B) 1 (C) 2 (D) Any natural number		
Sol.	(B) 1		1
	Assertion – Reason Based Questions **Direction :** Questions number **19** and **20** are Assertion and Reason based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below. (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true and Reason (R) is false. (D) Assertion (A) is false and Reason (R) is true.		

19.	**Assertion (A)** : In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$. **Reason (R)** : For any two events A and B, $P(A\|B) = \frac{P(A \cup B)}{P(B)}$		
Sol.	(C) Assertion (A) is true and Reason (R) is false.		1
20.	**Assertion (A)** : Lines given by $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$. **Reason (R)** : Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are perpendicular to each other if $\vec{b}_1 \cdot \vec{b}_2 = 0$.		
Sol.	(D) Assertion (A) is false and Reason (R) is true.		1
	SECTION B Q. Numbers 21 to 25 are very short answer questions of 2 marks each.		
21.(a)	Check whether function f(x) defined as $f(x) = \begin{cases} \frac{\|x-3\|}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ is continuous at $x = 3$ or not ?		
Sol.	$f(x) = \begin{cases} \frac{-(x-3)}{2(x-3)}, x < 3 \\ \frac{x-6}{6}, x \geq 3 \end{cases} \Rightarrow f(x) = \begin{cases} \frac{-1}{2}, x < 3 \\ \frac{x-6}{6}, x \geq 3 \end{cases}$ $f(3) = \frac{-1}{2}$ LHL = $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{-1}{2} = \frac{-1}{2}$ RHL = $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \frac{x-6}{6} = \frac{-1}{2}$	I II	1 ½

	LHL = RHL = $f(3)$, hence the function $f(x)$ is continuous at $x = 3$.	III	½
	OR		
21.(b)	(b) If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.		
Sol.	Differentiating both sides w.r.to x, we get $\sqrt{3}\left(2x + 2y\frac{dy}{dx}\right) = 4(x\frac{dy}{dx} + y)$ $\Rightarrow \frac{dy}{dx} = \frac{2y - \sqrt{3}x}{\sqrt{3}y - 2x}$ $\Rightarrow \left(\frac{dy}{dx}\right)_{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)} = \sqrt{3}$	I II III	1 ½ ½
22.	A room freshner bottle in the shape of an inverted cone sprays the perfume at regular intervals such that volume of the perfume in the bottle decreases at the steady rate of 1 mm³/min. Find the rate at which level of perfume is dropping at an instant when level of perfume in the bottle is 10 mm, if the semi-vertical angle of conical bottle is $\frac{\pi}{6}$.  		
Sol.	Let v, r, h, respectively, be the volume, radius and the height of perfume at any time t. $\frac{dv}{dt} = -1\ mm^3/min$ $\frac{r}{h} = tan\frac{\pi}{6} \Rightarrow r = \frac{h}{\sqrt{3}}$ $v = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\frac{h^3}{3}$ $\Rightarrow \frac{dv}{dt} = \frac{\pi}{9}3h^2\frac{dh}{dt}$	I II III	½ ½ ½

	$\Rightarrow \frac{dh}{dt} = -\frac{3}{\pi h^2}$ $\Rightarrow (\frac{dh}{dt})_{h=10} = -\frac{3}{100\pi}$ The level of perfume is dropping at the rate of $\frac{3}{100\pi}$ mm/min	IV	½
23.	Find the vector of magnitude 14 in the direction of $\overrightarrow{QP}$, where P and Q are the points (1, 3, 2) and (–1, 0, 8) respectively.		
Sol.	$\overrightarrow{QP} = 2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ $\lvert\overrightarrow{QP}\rvert = \sqrt{4+9+36} = 7$ The unit vector in the direction of $\overrightarrow{QP} = \frac{\overrightarrow{QP}}{\lvert\overrightarrow{QP}\rvert} = \frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} - \frac{6}{7}\hat{k}$ The required vector $= 14(\frac{\overrightarrow{QP}}{\lvert\overrightarrow{QP}\rvert}) = 4\hat{\imath} + 6\hat{\jmath} - 12\hat{k}$	I II III	½ ½ 1
24.	Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths.		
Sol.	$\vec{a} + \vec{b} = 4\hat{\imath} - 2\hat{\jmath} + 4\hat{k}, \vec{a} - \vec{b} = 2\hat{\imath} - 2\hat{\jmath}$ The vector representing one of its diagonals is $4\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ *or* $-(4\hat{\imath} - 2\hat{\jmath} + 4\hat{k})$ The vector representing the other diagonal is $2\hat{\imath} - 2\hat{\jmath}$ *or* $-(2\hat{\imath} - 2\hat{\jmath})$ The lengths of the two diagonals are $\sqrt{16+4+16} = 6$ and $\sqrt{4+4} = 2\sqrt{2}$	I II III	½ ½ 1
25. (a)	Simplify : $\tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right)$, $0 < x < \frac{\pi}{4}$.		
Sol.	Given expression $= tan^{-1}(\frac{1-tan2x}{1+tan2x})$ $= tan^{-1}\left[\tan\left(\frac{\pi}{4} - 2x\right)\right] = \frac{\pi}{4} - 2x$	I II	1 1
	OR		

25. (b)	$$\text{Evaluate : } \tan\left(\sin^{-1}1 - \cos^{-1}\left(-\frac{1}{2}\right)\right)$$		
Sol.	Given expression = $\tan\left(\frac{\pi}{2} - \frac{2\pi}{3}\right)$ $= \tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$	I II	1 1
	SECTION C Q. Numbers 26 to 31 are short answer questions of 3 marks each.		
26.	$$\text{Evaluate : } \int_0^1 x \tan^{-1}x \, dx.$$		
Sol.	$\int_0^1 x\tan^{-1}x dx = \left[\tan^{-1}x \times \frac{x^2}{2}\right]_0^1 - \int_0^1 \frac{1}{1+x^2} \times \frac{x^2}{2} dx$ $= \frac{\pi}{8} - \int_0^1 \frac{1+x^2-1}{1+x^2} \times \frac{1}{2} dx$ $= \frac{\pi}{8} - \frac{1}{2}\left[\int_0^1 dx - \int_0^1 \frac{1}{1+x^2} dx\right]$ $= \frac{\pi}{8} - \frac{1}{2}[x - \tan^{-1}x]_0^1$ $= \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8} = \frac{\pi}{4} - \frac{1}{2}$	I II III	1 1 1
27. (a)	$$\text{Find } \int \sqrt{\frac{x+2}{x-2}} \, dx$$		
Sol.	$I = \int \sqrt{\frac{x+2}{x-2}} dx = \int \sqrt{\frac{x+2}{x-2} \times \frac{x+2}{x+2}} dx = \int \frac{x+2}{\sqrt{x^2-4}} dx$ $= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-4}} dx + 2\int \frac{1}{\sqrt{x^2-4}} dx$	I II	1 1

	$I_1 = \frac{1}{2}\int \frac{2x}{\sqrt{x^2-4}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$ $[(x^2-4) = t \Rightarrow 2xdx = dt]$ $= \sqrt{t} = \sqrt{x^2-4}$ $I_2 = 2\int \frac{1}{\sqrt{x^2-4}}dx = 2log\left\|x+\sqrt{x^2-4}\right\|$ $I = I_1 + I_2 = \sqrt{x^2-4} + 2log\left\|x+\sqrt{x^2-4}\right\| + C$	III IV	1/2 1/2
	OR		
27. (b)	Find : $\int \frac{x^2}{(x^2+9)(x^2+16)}dx$		
Sol.	Put $x^2 = t$ to get $\frac{x^2}{(x^2+9)(x^2+16)} = \frac{t}{(t+9)(t+16)} = \frac{A}{t+9} + \frac{B}{t+16}$ Getting A$= \frac{-9}{7}, B = \frac{16}{7}$ Given integral $= \frac{-9}{7}\int \frac{1}{x^2+9}dx + \frac{16}{7}\int \frac{1}{x^2+16}dx$ $= \frac{-3}{7}tan^{-1}\left(\frac{x}{3}\right) + \frac{4}{7}tan^{-1}\left(\frac{x}{4}\right) + C$	I II III IV	½ 1 ½ 1
28.	If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ and $I_2 = \int_{-1/2}^{1/2} \|x\| dx$, then show that $I_1 - 4I_2 = 0$.		
Sol.	$I_1 = 2\int_0^{\frac{\pi}{4}} \frac{dx}{1+cos2x}$, $\left(\frac{1}{1+cos2x}\right.$ is an even function) $= 2\int_0^{\frac{\pi}{4}} \frac{dx}{2cos^2x} = \int_0^{\frac{\pi}{4}} sec^2xdx$ $= [tanx]_0^{\frac{\pi}{4}} = 1$ Note: Students attempting I_1 without the application of any property should be marked proportionately. $I_2 = 2\int_0^{\frac{1}{2}} \|x\|dx,$ ($\|x\|$ is an even function) $= 2\int_0^{\frac{1}{2}} xdx$	I II	1½ 1/2

	$= [x^2]_0^{\frac{1}{2}} = \frac{1}{4}$ $I_1 - 4I_2 = 0$	III IV	½ ½
29. (a)	Find the general solution of the following differential equation : $x^2 \frac{dy}{dx} = x^2 + xy + y^2$		
Sol.	Given differential equation is $\frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$ Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ The given differential equation becomes $x\frac{dv}{dx} = 1 + v^2$ $\Rightarrow \frac{dv}{1+v^2} = \frac{dx}{x}$ $\Rightarrow \int \frac{dv}{1+v^2} = \int \frac{dx}{x}$ $\Rightarrow tan^{-1}v = log\|x\| + C$ $\Rightarrow tan^{-1}\left(\frac{y}{x}\right) = log\|x\| + C$	I II III IV V	1/2 1/2 ½ 1 ½
	OR		
29. (b)	Find the particular solution of the differential equation $xy\frac{dy}{dx} = (x + 2)(y + 2)$, given that $y(1) = -1$.		
Sol.	Given differential equation is $\frac{y}{y+2}dy = \frac{(x+2)}{x}dx$ $\Rightarrow \int (1 - \frac{2}{y+2})dy = \int \left(1 + \frac{2}{x}\right)dx$ $\Rightarrow y - 2log\|y+2\| = x + 2log\|x\| + C$ or $y = x + 2log\|x(y+2)\| + C$. When $x = 1, y = -1 \Rightarrow C = -2$ The required particular solution is	I II	1 1½

<table>
<tr><td></td><td>$y - 2log|y+2| = x + 2log|x| - 2 \ \ or \ y = x + 2log|x(y+2)| - 2$</td><td>III</td><td>½</td></tr>
<tr><td>30.</td><td>Solve the following linear programming problem graphically :

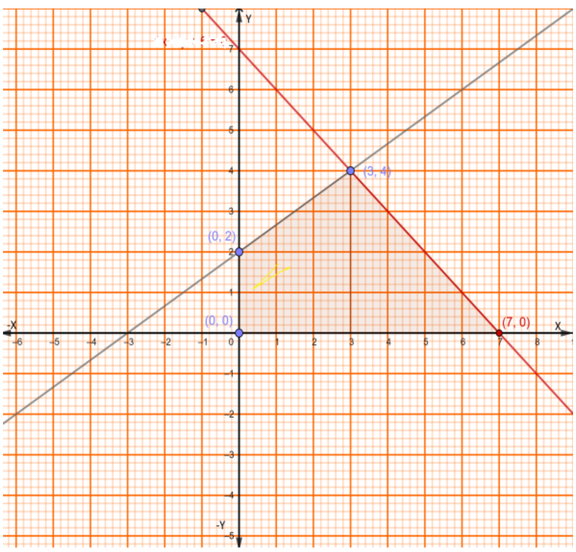
Minimize $\quad Z = 13x - 15y$

Subject to constraints

$x + y \leq 7,$

$2x - 3y + 6 \geq 0,$

$x \geq 0, y \geq 0$</td><td></td><td></td></tr>
<tr><td>Sol.</td><td>



<table>
<tr><td>Corner point</td><td>Value of $Z = 13x - 15y$</td></tr>
<tr><td>(0, 0)</td><td>0</td></tr>
<tr><td>(7, 0)</td><td>91</td></tr>
<tr><td>(3, 4)</td><td>-21</td></tr>
<tr><td>(0, 2)</td><td>-30</td></tr>
</table>
The minimum value of Z is -30 at $x = 0, y = 2$</td><td>I

II

III</td><td>1½ (Correct graph and shading)

1

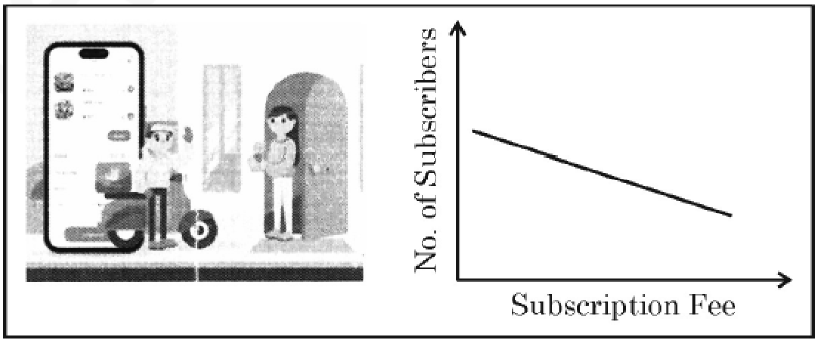
½</td></tr>
</table>

31.(a)	Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball.		
Sol.	Let E_1: The number appearing on the die < 3 E_2: The number appearing on the die ≥ 3 A: Red ball is drawn	I	1/2
	$P(E_1) = \frac{2}{6}, P(A/E_1) = \frac{3}{7}, P(E_2) = \frac{4}{6}, P(A/E_2) = \frac{8}{14}$	II	1
	The required probability = $P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$ $= \frac{2}{6} \times \frac{3}{7} + \frac{4}{6} \times \frac{8}{14}$	III	1
	$= \frac{11}{21}$	IV	½
	OR		
31.(b)	The probability of simultaneous occurrence of atleast one of the two events X and Y is a. If the probability that exactly one of the events X, Y occurs is b, prove that P(X′) + P(Y′) = 2 – 2a + b.		
Sol.	$P(X \cup Y) = a$	I	½
	$P(X \cup Y) - P(X \cap Y) = b$ Hence, $P(X \cap Y) = a - b$	II	1
	$P(X') + P(Y') = [1 - P(X)] + [1 - P(Y)]$ $= 2 - [P(X) + P(Y)]$ $= 2 - [P(X \cup Y) + P(X \cap Y)]$ $= 2 - [a + (a - b)]$	III	1

	$= 2 - 2a + b$	IV	1/2
	SECTION D Q. Numbers 32 to 35 are long answer questions of 5 marks each.		
32.(a)	A relation R is defined on Z, the set of integers, as $R = \{(x, y) : \|x - y\|$ is divisible by a prime number 'p', $x, y \in Z\}$ check whether R is an equivalence relation or not.		
Sol.	Let $x \in Z$. Then $\|x - x\| = 0$, which is divisible by a prime number p. Hence, $(x, x) \in R$. Thus, R is reflexive. Let $x, y \in Z$ such that $(x, y) \in R$. Then $\|x - y\|$ is divisible by the prime number p. $\Longrightarrow \|y - x\|$ is divisible by the prime number p as $\|x - y\| = \|y - x\|$ Hence, $(y, x) \in R$. Thus, R is symmetric. Let $x, y, z \in Z$ such that $(x, y), (y, z) \in R$. Then $\|x - y\|$ is divisible by the prime number p and $\|y - z\|$ is divisible by the prime number p. $\Longrightarrow x - y$ is divisible by the prime number p and $y - z$ is divisible by the prime number p. $\Longrightarrow x - y + y - z$ is divisible by the prime number p $\Longrightarrow x - z$ is divisible by the prime number p $\Longrightarrow \|x - z\|$ is divisible by the prime number p Hence, $(x, z) \in R$. Thus, R is transitive. Since, R is reflexive, symmetric and transitive, therefore, R is an equivalence relation.	I II III IV	1½ 1½ 1½ ½
	OR		
32. (b)	A function $f : R - \left\{\frac{3}{5}\right\} \longrightarrow R - \left\{\frac{3}{5}\right\}$ is defined as $f(x) = \frac{3x+2}{5x-3}$. Show that f is one-one and onto.		
Sol.	Let $x_1, x_2 \in R - \left\{\frac{3}{5}\right\}$ such that $f(x_1) = f(x_2)$		

	$\Rightarrow \dfrac{3x_1+2}{5x_1-3} = \dfrac{3x_2+2}{5x_2-3}$ $\Rightarrow 15x_1x_2 - 9x_1 + 10x_2 - 6 = 15x_1x_2 + 10x_1 - 9x_2 - 6$ $\Rightarrow x_1 = x_2$ Hence, f is one-one. Let $y \in R - \left\{\frac{3}{5}\right\}$ (Codomain). Then $f(x) = y$ $or, \frac{3x+2}{5x-3} = y \Rightarrow x = \frac{3y+2}{5y-3} \in$ Domain. Hence, for every $y \in R - \left\{\frac{3}{5}\right\}$ $(codomain)$, there exists $x = \frac{3y+2}{5y-3} \in R - \left\{\frac{3}{5}\right\}$ $(domain)$ such that $f\left(\frac{3y+2}{5y-3}\right) = y$ Thus, f is onto.	I II	2½ 2½
33. (a)	If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations : $-2y + z = 7,\ 2x - y - z = 8,\ x - 2y = 10$		
Sol.	$\|A\| = -2(2) + 1(3) = -1 \neq 0$ $adjA = \begin{bmatrix} -2 & -1 & -3 \\ -2 & -1 & -2 \\ 3 & 2 & 4 \end{bmatrix}$ $A^{-1} = \frac{1}{\|A\|} adjA = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ -3 & -2 & -4 \end{bmatrix}$ Given system of equations is equivalent to $A^T X = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}$ $X = (A^T)^{-1}B = (A^{-1})^T B$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 2 & -3 \\ 1 & 1 & -2 \\ 3 & 2 & -4 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ $\therefore x = 0, y = -5, z = -3$	I II III IV V	1 1½ ½ ½ 1½
	OR		

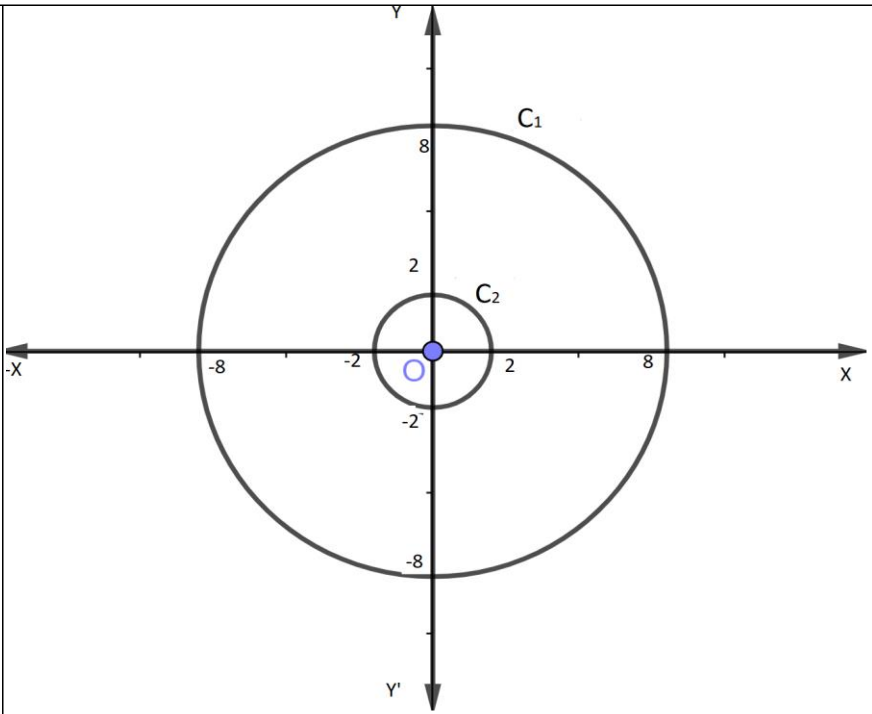
33. (b)	If $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix, then find all values of x where $x \in \left[0, \frac{\pi}{2}\right]$.		
Sol.	Given matrix is singular, hence, its determinant = 0, $i.e., \begin{vmatrix} 3 & -1 & sin3x \\ -7 & 4 & cos2x \\ -11 & 7 & 2 \end{vmatrix} = 0$ $\Rightarrow 2cos2x + sin3x = 2$ $\Rightarrow 4sin^3x + 4sin^2x - 3sinx = 0$ $\Rightarrow sinx(4sin^2x + 4sinx - 3) = 0$ $\Rightarrow sinx(2sinx - 1)(2sinx + 3) = 0$ Solving to get $x = 0, \frac{\pi}{6}$	I II III IV V	1 1 1 1 1
34.	If $x = \cos t$, $y = \cos mt$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$.		
Sol.	$\frac{dx}{dt} = -sint, \frac{dy}{dt} = -m \times sin(mt)$ $\frac{dy}{dx} = \frac{m \times sin(mt)}{sint}$ $or\ sint\frac{dy}{dx} = m \times sin(mt)$ $\Rightarrow sint\frac{d^2y}{dx^2} + \frac{dy}{dx}cost \times \frac{dt}{dx} = m^2\cos(mt) \times \frac{dt}{dx}$ $\Rightarrow (1 - cos^2t)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -m^2\cos(mt)$ $\Rightarrow (1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2\mathrm{y} = 0$	I II III IV	1 1 2 1
35.	Check whether the lines given by $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are parallel or not. If parallel, find the distance between them, otherwise find their point of intersection, if the lines are intersecting.		
Sol.	Drs of first line are < 2, 3, 4 >		

	Drs of second line are < 5, 2, 1 >	I	1
	Drs of the two lines are not proportional. Hence, the lines are not parallel.	II	1
	Any point on the first line is $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$		
	Any point on the second line is $(5\mu + 4, 2\mu + 1, \mu)$	III	1
	For the lines to intersect, we must have some λ and μ, for which these coordinates must coincide, i.e., we must have $2\lambda + 1 = 5\mu + 4, 3\lambda + 2 = 2\mu + 1, 4\lambda + 3 = \mu$, for some λ and μ		
	The first two equations, when solved, give us $\lambda = -1$ and $\mu = -1$. These values satisfy the third equation. Hence, the lines intersect.	IV	1
	The point of intersection is (-1, -1, -1)	V	1
	SECTION E This section (Q. 36 to 38) has 3 case study-based questions of 4 marks each.		
36.	An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹ 300 per subscriber for unlimited free deliveries.  The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹ 1, ten subscribers will discontinue. Assume that the company increased the annual fee by ₹ x.		

	Based on the given information, answer the following questions : (i) How many subscribers will discontinue after an increase of ₹ x in annual fee ? (ii) If R(x) denotes the total revenue collected after the increase of ₹ x in subscription fee, express R(x) as a function of x. (iii) (a) Find the value of x for which R(x) is maximum. **OR** (iii) (b) Find the sub-intervals of (0, 5000) in which R(x) is increasing and decreasing.		
Sol.(i)	$10x$	I	1
Sol.(ii)	$R(x) = (5000 - 10x)(300 + x)$	I	1
Sol.(iii) (a)	$R'(x) = 2000 - 20x$ $R'(x) = 0 \Longrightarrow x = 100$ $R''(x) = -20 < 0$ at $x = 100$ At $x = 100, R(x)$ is maximum	I II III IV	½ ½ ½ ½
	OR		
Sol.(iii) (b)	$R'(x) = 2000 - 20x$ $R'(x) = 0 \Longrightarrow x = 100$ In (0, 100), $R'(x) > 0$.Therefore, $R(x)$ is increasing in (0, 100) or (0, 100] In (100, 5000), $R'(x) < 0$.Therefore, $R(x)$ is decreasing in (100, 5000) or [100, 5000)	I II III IV	1/2 1/2 1/2 1/2

37.	In an online jackpot, there is one first prize of ₹ 3,00,000, two second prizes of ₹ 2,00,000 each and three third prizes of ₹ 50,000 each.  A total of 1,00,000 jackpot tickets each costing ₹ 100 were sold there by raising a fund of ₹ 1,00,00,000. Rohan bought one ticket. Based on given information, answer the following questions : (i) What are the possible amounts, the person can win ? (ii) (a) What is the probability that the person wins atleast ₹ 2,00,000 ? **OR** (ii) (b) What is the probability that the person does not win any amount ? (iii) In another jackpot, Rohan also bought a ticket having a prize money of ₹ 5,00,000. The chances of winning the jackpot are 1 in 1,00,000. Find the probability that on exactly one of tickets he wins the jackpot.		
Sol.(i)	₹ 300000, ₹ 200000, ₹ 50000	I	1
Sol.(ii) (a)	Required probability = $\frac{3}{100000}$	I	2
	OR		
Sol.(ii) (b)	Required probability = $1-\frac{6}{100000}=\frac{99994}{100000}\ or\ \frac{49997}{50000}$	I	2
Sol.(iii)	P (Exactly on one of the tickets Rohan wins the jackpot) = P [(He wins first and he loses second) or (He loses first and wins second)] $=\frac{6}{100000}\times\frac{99999}{100000}+\frac{99994}{100000}\times\frac{1}{100000}=\frac{699988}{10^{10}}\ or\ \frac{174997}{2500000000}$	I	1

38.	Roundabouts are often made on busy roads to ease the traffic and avoid red lights.  One such round-about is made such that equation representing its boundary is given by C_1 ; $x^2 + y^2 = 64$. There is a circular pond with a fountain in the middle of the roundabout whose equation is given by $C_2 : x^2 + y^2 = 4$. Based on the given information, answer the following questions : (i) Represent the given equations C_1 and C_2 with the help of a diagram. (ii) Express y as a function of x, ($y = f(x)$), for both C_1 an C_2. (iii) (a) Using integration find the area of region covered by the roundabout. **OR** (iii) (b) Using integration, find the area of region covered by circular pond.		

Sol.(i)		I	1
Sol.(ii)	Circle C_1: $y = \sqrt{64 - x^2}$ or $y = -\sqrt{64 - x^2}$ Circle C_2: $y = \sqrt{4 - x^2}$ or $y = -\sqrt{4 - x^2}$	I II	½ ½
Sol.(iii) (a)	Required area = $4 \times \int_0^8 \sqrt{64 - x^2}\, dx$ $= 2[x\sqrt{64 - x^2} + 64sin^{-1}\frac{x}{8}]_0^8 = 64\pi$	I II	1 1
	OR		
Sol.(iii) (b)	Required area = $4 \times \int_0^2 \sqrt{4 - x^2}\, dx$ $= 2[x\sqrt{4 - x^2} + 4sin^{-1}\frac{x}{2}]_0^2 = 4\pi$	I II	1 1